Calculus 1 FINAL EXAM LEAKED!

1. Evaluate the limits.

   (a) $\displaystyle\lim_{x \to 4} \frac{x^2 - 7x + 12}{x^3 - 16x}$

   (b) $\displaystyle\lim_{y \to 3} \frac{\frac{1}{y} - \frac{1}{3}}{y - 3}$

   (c) $\displaystyle\lim_{x \to 4^+} \frac{3x + 1}{x(x - 4)}$

   (d) $\displaystyle\lim_{x \to 4^-} \frac{3x + 1}{x(x - 4)}$

2. Show that the following equation has a root in the interval $[1, 4]$.

$$\frac{2}{x} - x + \sqrt{x} = 0$$

3. (a) State the limit definition of the derivative of f.

   (b) Use the limit definition of the derivative to find the derivative of $f(x) = 3x - 8$.

4. Find an expression for the area under the curve $y = x$ from 0 to 3 as a limit, then evaluate the limit. The following identity may be helpful:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}.$$

5. Let $y = \displaystyle\int_{x^2}^{3x+2} \cos^2 t + 1 \; dt$. Find y'.

6. Evaluate the integrals.

(a) $\displaystyle\int \frac{(x+1)(x+2)}{\sqrt{x}} \; dx$.

(b) $\displaystyle\int \frac{4 - 5\sin^3(x)}{\sin^2(x)} \; dx$.

(c) $\displaystyle\int_{-1/3}^{10} \sqrt[5]{2+3x} \; dx$

7. Find the absolute extreme values of the function $f(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 + 2x$ on the interval $[0, 2]$.

8. State why the function $f(x) = 2x^2 - 4x + 5$ satisfies the Mean Value Theorem on the interval $[-1, 3]$, then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

9. Find the critical numbers of the function $f(x) = x^{4/5}(5 - x)$.

10. Find the limit or show that it does not exist.

$$\lim_{x\to\infty} \left(\sqrt{x^2 + 5x} - x\right)$$

11. Use the guidelines to sketch the graph of

$$f(x) = \frac{2x^2}{x^2 - 25}.$$

Include: Domain, Intercepts, Symmetry, Asymptotes, Intervals of increase or decrease, local max and min, concavity and points of infection, and sketch.

$$f'(x) = \frac{-100x}{(x^2 - 25)^2}, \; f''(x) = \frac{100(3x^2 + 25)}{(x^2 - 25)^3}.$$

12. Compute the derivatives of the following functions.

(a) $f(\theta) = 2\theta \sin\theta \cos\theta$

(b) $h(x) = \dfrac{3x+1}{\cot x}$

(c) $g(t) = \left(\sqrt{t + \cos(t^2)}\right)^3$

13. An object is thrown vertically. Its height in meters as a function of time is given by

$$s(t) = -4.9t^2 + 19.6t + 58.8.$$

Time is measured in seconds.

(a) Calculate the velocity, $v(t)$.

(b) Find the maximum height that the object reaches.

(c) Find the time when the object hits the ground.

(d) For what positive times is the object slowing down?

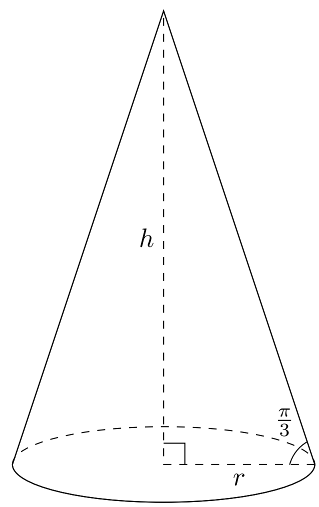
(e) For what positive times is the acceleration negative?

14. Find the equation of the tangent line to $x^3 + 2y^3 = 5xy$ at the point $(2, 1)$.

15. Use a linear approximation to approximate $\sqrt[3]{8.012}$

16. The sum of two positive numbers is 16. What is the smallest possible value of the sum of their squares?

17. Gravel is being dumped from a conveyor belt at a rate of 35 ft^3/min, and its coarseness is such that it forms a pile in the shape of a cone. The sides of the pile always make an angle of $\frac{\pi}{3}$ with the ground. How fast is the height of the pile increasing when the pile is 15 ft high? [The volume of a cone is $\frac{1}{3}\cdot$(area of the base)$\cdot$(height).]



18. If $f''(t) = \sqrt[3]{t} - 3t^2, f(0) = 2$, and $f(1) = 2$, find $f(t)$.