Topics you should understand (the ones in bold are extra important):

- **Absolute Maximum and Absolute Minimum**
- **Local Maximum and Local Minimum**
- **Critical Numbers**
- **Increasing/Decreasing Test**
- **First Derivative Test**
- **Concavity Test**
- **Inflection Point**
- **Second Derivative Test**
- **Curve Sketching by Hand**
- **Optimization**
- **Linear Approximation**
- **Newton's Method**
- **Implicit Differentiation**
- **Related Rates**

Practice Problems:

1. If $x^2 + xy + y^3 = 1$, find the value of y'' at the point where $x = 1$.
2. Let $y = \arcsin(3x^2 - 6x + 4)$. Use implicit differentiation to express y' as a function of x. Your answer should not have any trig functions in it.
3. Let $y = \operatorname{arccot}(x^2 + x)$. Use implicit differentiation to express y' as a function of x. Your answer should not have any trig functions in it.

   Sol: Click here to watch the solution!
4. Let $y = \arccos(x)$. Use implicit differentiation to express y' as a function of x. Your answer should not have any trig functions in it.

   Sol: Click here to watch the solution!
5. Use implicit differentiation to find an equation of the tangent line to the curve at the given point.

$$y\sin(2x) = x\cos(2y), \qquad (\frac{\pi}{2}, \frac{\pi}{4})$$

   Sol: Click here to watch the solution!

6. The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3 m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?

7. If a snowball melts so that its surface area decreases at a rate of 1 square cm/minute, find the rate at which the radius decreases when the radius is 5 cm.

   Sol: Click here to watch the solution!

8. A spotlight on the ground shines on a wall 12 m away. If a man 2 m tall walks from the spotlight toward the building at a speed of 1.6 m/s, how fast is the length of his shadow on the building decreasing when he is 4 m from the building?

9. At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4:00 PM?

   Sol: Click here to watch the solution!

10. The area of a triangle with sides of lengths a and b and contained angle θ is

$$A = \frac{1}{2}ab\sin\theta$$

    If $a = 2$, $b = 3$, and θ increases at a rate of 0.2 rad/min, how fast is the area increasing when $\theta = \pi/3$? Here a and b are constants.

11. A street light is mounted at the top of a 20-ft-tall pole. A lady 5 ft tall walks away from the pole along a straight path. When the lady is 30 feet from the pole, the tip of her shadow is moving at a rate of 10 ft/sec. How fast is the lady walking?

    Sol: Click here to watch the solution!

12. A particle is moving along a hyperbola $xy = 8$. As it reaches the point $(4, 2)$, the y-coordinate is decreasing at a rate of 3 cm/s. How fast is the x-coordinate of the point changing at that instant?

    Sol: Click here to watch the solution!

13. Use a linear approximation to estimate the value of $\dfrac{1}{4.002}$.

14. Use a linear approximation to estimate the value of $\sqrt{4.02}$.

    Sol: Click here to watch the solution!

15. Use a linear approximation to estimate the value of $\sqrt[3]{7.95}$.

16. Use a linear approximation to estimate the value of 1.999^4.

    Sol: Click here to watch the solution!

17. Approximate the value of $\sqrt[3]{6}$ by using Newton's Method.

    Sol: Click here to watch the solution!

18. Sketch the graph of a function f that is continuous on $[1, 5]$ and has the given properties: absolute maximum at 2, absolute minimum at 5, 4 is a critical number but there is no local maximum or minimum there.

19. Find the critical number of the functions $g(x) = x^4 + x^3 + x^2 + 1$.

20. Find the critical numbers of $f(x) = 2\cos(x) + \sin(2x)$ on the interval $[0, 2\pi]$.
    Sol: Click here to watch the solution!

21. Find the absolute maximum and absolute minimum values of f on the given interval:
$$f(x) = 3x^4 - 4x^3 - 12x^2 + 1, \qquad [-2, 3].$$

22. Find the absolute maximum and absolute minimum values of f on the given interval:
$$f(x) = x^3 - 6x^2 + 5, \qquad [-3, 5].$$
    Sol: Click here to watch the solution!

23. Find the absolute maximum and absolute minimum values of f on the given interval:
$$f(x) = x + \frac{1}{x}, \qquad [0.2, 4].$$

24. Find the absolute maximum and absolute minimum values of f on the given interval:
$$f(x) = \frac{1 - x}{x^2 + 3x}, \qquad [1, 4].$$
    Sol: Click here to watch the solution!

25. Find the local maximum and minimum values of $f(x) = (x^2 - 2x)^2$.
    Sol: Click here to watch the solution!

26. Find the local maximum and minimum values of $f(x) = 3x^{\frac{1}{3}} - x$.
    Sol: Click here to watch the solution!

27. Find the local maximum and minimum values of $f(x) = x\sqrt{x + 3}$.
    Sol: Click here to watch the solution!

28. Use the Second Derivative Test to analyze the critical points of $f(x) = x^5 - 5x^4$.
    Sol: Click here to watch the solution!

29. Sketch the graph of a function f that is continuous on its domain, $(-6, 5]$, and where $f(0) = 1$, $f'(0) = 0$, $f'(2)$ does not exist, $\lim_{x\to 5^-} f(x) = 4$, $\lim_{x\to -5^+} f(x) = \infty$.
    Sol: Click here to watch the solution!

30. Let $f(x) = x^3 - 3x^2 - 9x + 4$.
    (a) Find the intervals on which f is increasing or decreasing.
    (b) Find the local maximum and minimum values of f.
    (c) Find the intervals of concavity and the inflection points.

31. Let $f(x) = \dfrac{x}{x^2 + 1}$.

(a) Find the intervals on which f is increasing or decreasing.

(b) Find the local maximum and minimum values of f.

(c) Find the intervals of concavity and the inflection points.

32. Sketch the graph of a function that satisfies all of the given conditions:

- $f'(0) = f'(4) = 0$,
- $f'(x) = 1$ if $x < 1$,
- $f'(x) < 0$ if $-1 < x < 0$, or $2 < x < 4$, or $x > 4$,
- $\lim_{x \to 2^-} f(x) = \infty$,
- $\lim_{x \to 2^+} f(x) = -\infty$,
- $f''(x) > 0$ if $-1 < x < 2$ or $2 < x < 4$,
- $f''(x) < 0$ for $x > 4$.

Sol: Click here to watch the solution!

33. Sketch the graph of a function that satisfies all of the given conditions:

- $f(1) = f'(1) = 0$,
- $\lim_{x \to 2^+} f(x) = \infty$,
- $\lim_{x \to 2^-} f(x) = -\infty$,
- $\lim_{x \to 0} f(x) = -\infty$,
- $\lim_{x \to -\infty} f(x) = \infty$,
- $\lim_{x \to \infty} f(x) = 0$,
- $f''(x) > 0$ for $x > 2$,
- $f''(x) < 0$ for $x < 0$ and $0 < x < 2$.

34. Sketch the graph of the function $f(x) = x^{\frac{1}{3}}(x + 4)$ without the use of a graphing calculator.

Sol: Click here to watch the solution!

35. Let $f(x) = \dfrac{x^3}{x^3 + 1}$. Find the following:

(a) the domain of f

(b) the intercepts

(c) all vertical and horizontal asymptotes

(d) intervals of increase or decrease

(e) local maximum and minimum values

(f) concavity and points of inflection

Use parts (a) - (f) to sketch the graph of f.

36. Find two numbers whose difference is 100 and whose product is a minimum.

37. Find an equation of the line through the point $(3, 5)$ that cuts off the least area from the first quadrant.

    Sol: Click here to watch the solution!

38. A farmer wants to fence in an area of 1.5 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. How can he do this so as to minimize the cost of the fence?

39. if you are offered one slice of pizza (in other words, a sector of a circle) and the slice must have a perimeter of 32 inches, what diameter pizza will reward you with the largest slice?

    Sol: Click here to watch the solution!

40. A box with a square base and open top must have a volume of 32000 cm^3. Find the dimensions of the box that minimize the amount off material used.

41. Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

    Sol: Click here to watch the solution!

42. A farmer wants to fence in a rectangular plot of land adjacent to the north wall of his barn. No fencing is needed along the barn, and the fencing along the west side of the plot is shared with a neighbor who will split the cost of that portion of the fence. If the fencing costs \$20 per linear foot to install and the farmer is not willing to spend more than \$5000, find the dimensions for the plot that would enclose the most area.

43. Find the point on the ellipse $9x^2 + y^2 = 9$ that is farthest from the point $(1, 0)$.

    Sol: Click here to watch the solution!

44. Find the point on the curve $y + 2x = 4$ that is closest to the point $(5, 0)$.